

Implications of Planck results for models with local type non-Gaussianity

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Abstract

We discuss implications of Planck results for models with local type non-Gaussianity. In light of the recent results of the Planck satellite, we constrain model parameters of several representative models and give the prediction of trispectrum, in particular, g_{NL} . We also consider interesting possibilities that trispectrum appears as the first signature of the non-Gaussianities of the curvature perturbations, that is, f_{NL} is small while g_{NL} can be significantly large.

1 Introduction

Very recently, the Planck mission has released data from the first 15.5 months of Planck operations for cosmic microwave background (CMB) anisotropies [1]. They determined cosmological parameters with unprecedented accuracy such as the baryon, the dark matter, and the dark energy densities, which strongly support the so-called concordance model of cosmology [2]. They also gave strong constraints on primordial curvature perturbations. The spectral index $n_s = 0.9603 \pm 0.0073$ (68%CL) [3] significantly deviates from unity, supporting the slow-roll inflation paradigm. Any deviations from Gaussianities of primordial curvature perturbations are not found. In particular, the local type of f_{NL} is now strongly constrained as $-8.9 < f_{\text{NL}} < 14.3$ at two sigma level [4], which rules out a lot of light field models predicting large local type non-Gaussianities.

In fact, one may wonder if light field models such as the curvaton [5–7] and the modulated reheating [8, 9] scenarios might be excluded because they are often claimed to generate large non-Gaussianities. However, this is not the case. For example, as is well known, in the curvaton scenario with a quadratic potential, the local type of f_{NL} is given by [10, 11]

$$f_{\text{NL}} = \frac{5}{4r} - \frac{5}{3} - \frac{5r}{6}. \quad (1)$$

Here r is roughly the fraction of the curvaton energy density at the curvaton decay and is defined as

$$r = \left. \frac{3\rho_\sigma}{3\rho_\sigma + 4\rho_r} \right|_{\text{decay}}, \quad (2)$$

where ρ_σ is the curvaton energy density and ρ_r is the radiation energy density. The constraint $-8.9 < f_{\text{NL}} < 14.3$ (95% CL) is equivalent to $0.08 < r (\leq 1)$ (95% CL), which rules out the curvaton model with small r ^{#1}. However, to be fair, a natural value of r without fine-tuning is unity because the curvaton easily dominates the energy density of the Universe since the curvaton behaves like matter while the other components behave as radiation. Such a value of $r (= 1)$ yields $f_{\text{NL}} = -5/4$, which is still allowed by the recent Planck data. Thus, a simple and natural model of the curvaton is still viable.

In the same way, the modulated reheating scenario predicts f_{NL} as [12, 13]

$$f_{\text{NL}} = 5 \left(1 - \frac{\Gamma \Gamma_{\sigma\sigma}}{\Gamma_\sigma^2} \right), \quad (3)$$

where Γ is the inflaton decay rate depending on the modulus σ , $\Gamma_\sigma = \partial\Gamma/\partial\sigma$, $\Gamma_{\sigma\sigma} = \partial^2\Gamma/\partial\sigma^2$, and the inflaton is assumed to oscillate around its minimum with a quadratic potential. The constraint $-8.9 < f_{\text{NL}} < 14.3$ (95% CL) leads to $-1.9 < \Gamma\Gamma_{\sigma\sigma}/\Gamma_\sigma^2 < 2.8$ (95% CL), which suggests that the second derivative of Γ with respect to σ is significantly

^{#1}Here, we simply adopt the Planck constraint on f_{NL} to the constraint on the model parameter r . However, a direct likelihood analysis of r with adopting a prior $0 < r < 1$ performed by Planck team gives the constraint $0.15 \leq r$ (95% CL) [4]. This is the reason why the constraint by the Planck team is a bit different from ours. We thank Nicola Bartolo for the clarification on this point.

suppressed. Thus, as long as Γ linearly depends on the modulus σ , f_{NL} is predicted to be 5, which is still allowed by the recent Planck data.

Thus the light field models are still viable. After excluding the model parameters inconsistent with Planck data, we end up with the light field models that generically yield $f_{\text{NL}} = \mathcal{O}(1)$. Given that the standard inflation model, in which the inflaton itself is responsible for the curvature perturbations, predicts $\mathcal{O}(0.01)$ [14, 15], it is still important. Then, it is still important to detect order of unity f_{NL} for discriminating light field models from the standard inflation model though, in this case, we have to take into account the intrinsic CMB bispectrum $f_{\text{NL}} \lesssim 1$ coming from the second order effects of the evolution of the curvature perturbations [16, 17].

In this paper, we first discuss how much light field models are constrained according to the Planck data. As explained above, both of the curvaton and the modulated scenarios are still allowed, and, in some sense, the constrained value of f_{NL} is reasonable to avoid fine-tuning. Then, we give constraints on model parameters of light field models including the curvaton and the modulated scenarios and predict the trispectrum, in particular g_{NL} , for each model.

Next, we are going to pursue another interesting possibility. Though the recent data of the Planck satellite claims that bispectrum of the curvature perturbations is small, it does not necessarily imply that the non-Gaussianities of the curvature perturbations are insignificant because such non-Gaussianities might first appear on their trispectrum. In the latter half of this paper, such a possibility will be discussed in detail. Actually, for example, if we consider the modulated reheating scenario and its modulus also behaves like a curvaton, f_{NL} can be almost canceled and be small, but g_{NL} still can be large. We are going to discuss such possibilities and investigate which combination of model parameters can realize such possibilities.

The organization of the paper is as follows. In the next section, we constrain the model parameters of each light field model based on the recent Planck results and give the predictions of the trispectrum g_{NL} , whose observations are essentially important for pinning down the model. In Sec III. we discuss models, in which f_{NL} is small, actually, within the constraints given by the Planck satellite, but g_{NL} can be large as the first signal of the non-Gaussianities of the curvature perturbations. Detailed explanations of why g_{NL} can be large while keeping f_{NL} small for such models are given. The final section is devoted to summary of this paper.

2 Light field models

During inflation, there can be light fields other than the inflaton. They acquire quantum fluctuations during inflation, which can be converted to the curvature perturbations. Though a lot of conversion mechanisms have been proposed [18], the δN formalism [19–23] enables us to make a systematic treatment to evaluate the final curvature perturbations. According to the δN formalism, the superhorizon curvature perturbations at the final time

$t = t_f$ can be easily estimated by

$$\zeta(t_f) = N_a \delta\varphi_*^a + \frac{1}{2} N_{ab} \delta\varphi_*^a \delta\varphi_*^b + \frac{1}{6} N_{abc} \delta\varphi_*^a \delta\varphi_*^b \delta\varphi_*^c, \quad (4)$$

where t_* is the time shortly after the horizon exit, a subscript a, b , and c represents a light field, $N_a = \partial N / \partial \varphi_*^a$, and so on. Here, $\delta\varphi_*^a$ represents a field fluctuation evaluated at $t = t_*$ and is assumed to be Gaussian. Then, the bispectrum and the trispectrum of the curvature perturbations are characterized only by the three parameters, $f_{\text{NL}}^{\text{local}}$, $g_{\text{NL}}^{\text{local}}$, and $\tau_{\text{NL}}^{\text{local}}$ as follows,

$$\begin{aligned} \langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \zeta_{\vec{k}_3} \rangle &= (2\pi)^3 B_\zeta(k_1, k_2, k_3) \delta(\vec{k}_1 + \vec{k}_2 + \vec{k}_3), \\ \langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \zeta_{\vec{k}_3} \zeta_{\vec{k}_4} \rangle &= (2\pi)^3 T_\zeta(k_1, k_2, k_3, k_4) \delta(\vec{k}_1 + \vec{k}_2 + \vec{k}_3 + \vec{k}_4), \end{aligned} \quad (5)$$

where

$$B_\zeta(k_1, k_2, k_3) = \frac{6}{5} f_{\text{NL}}^{\text{local}} (P_\zeta(k_1)P_\zeta(k_2) + P_\zeta(k_2)P_\zeta(k_3) + P_\zeta(k_3)P_\zeta(k_1)), \quad (6)$$

$$\begin{aligned} T_\zeta(k_1, k_2, k_3, k_4) &= \tau_{\text{NL}}^{\text{local}} (P_\zeta(k_{13})P_\zeta(k_3)P_\zeta(k_4) + 11 \text{ perms.}) \\ &\quad + \frac{54}{25} g_{\text{NL}}^{\text{local}} (P_\zeta(k_2)P_\zeta(k_3)P_\zeta(k_4) + 3 \text{ perms.}), \end{aligned} \quad (7)$$

with $k_{13} = |\vec{k}_1 + \vec{k}_3|$. These three parameters are easily evaluated at the tree level according to the δN formalism,

$$\begin{aligned} \frac{6}{5} f_{\text{NL}}^{\text{local}} &= \frac{N_a N_b N^{ab}}{(N_c N^c)^2}, \\ \tau_{\text{NL}}^{\text{local}} &= \frac{N_a N_b N^{ac} N_c^b}{(N_d N^d)^3}, \\ \frac{54}{25} g_{\text{NL}}^{\text{local}} &= \frac{N_{abc} N^a N^b N^c}{(N_d N^d)^3}. \end{aligned} \quad (8)$$

Below we omit the suffix “local” for simplicity. There is no general relation between f_{NL} and g_{NL} because g_{NL} depends on the third derivative of N in addition to the first one. On the other hand, there is a general inequality between f_{NL} and τ_{NL} [24],

$$\tau_{\text{NL}} \geq \left(\frac{6}{5} f_{\text{NL}} \right)^2, \quad (9)$$

When only one source (field) contributes to the curvature perturbations, the equality must hold. On the other hand, when there are multiple sources, the equality can be violated. In the following analysis, we concentrate on single source case for simplicity. In this case, τ_{NL} is completely determined by f_{NL} as $\tau_{\text{NL}} = 36 f_{\text{NL}}^2 / 25$. Therefore, g_{NL} is a key observable quantity to discriminate light field models. Below, we constrain model parameters of light field models and give the prediction for g_{NL} based on the recent Planck satellite results. The extension of our discussions to multiple source case is straightforward and, generally speaking, the allowed region of the model parameters is widened.

2.1 Curvaton

In the curvaton model, the curvaton fluctuations are converted into the curvature perturbations when the curvaton decays into relativistic degrees of freedom, which occurs after inflation. The important quantities determining the resultant curvature perturbation are r representing the curvaton fraction to the total energy density and σ_{osc} , the curvaton value when the curvaton starts oscillations. If a curvaton potential deviates from a quadratic form, σ_{osc} generally depends on the curvaton value σ_* at the time of horizon crossing. The non-linearity parameters from the curvaton are given by [25]

$$\frac{6}{5}f_{\text{NL}} = \frac{3}{2r} \left(1 + \frac{\sigma_{\text{osc}}\sigma_{\text{osc}}''}{\sigma_{\text{osc}}'^2} \right) - 2 - r, \quad (10)$$

$$\begin{aligned} \frac{54}{25}g_{\text{NL}} = & \frac{9}{4r^2} \left(\frac{\sigma_{\text{osc}}^2\sigma_{\text{osc}}'''}{\sigma_{\text{osc}}'^3} + 3\frac{\sigma_{\text{osc}}\sigma_{\text{osc}}''}{\sigma_{\text{osc}}'^2} \right) - \frac{9}{r} \left(1 + \frac{\sigma_{\text{osc}}\sigma_{\text{osc}}''}{\sigma_{\text{osc}}'^2} \right) \\ & + \frac{1}{2} \left(1 - 9\frac{\sigma_{\text{osc}}\sigma_{\text{osc}}''}{\sigma_{\text{osc}}'^2} \right) + 10r + 3r^2, \end{aligned} \quad (11)$$

where $\sigma_{\text{osc}}' \equiv \partial\sigma_{\text{osc}}/\partial\sigma_*$ etc. Though the relation between σ_{osc} and σ_* is nontrivial in general, σ_{osc} has a linear dependence on σ_* for a quadratic potential of the curvaton. In this case ($\sigma_{\text{osc}}'' = \sigma_{\text{osc}}''' = 0$), the non-linear parameters reduce to

$$\frac{6}{5}f_{\text{NL}} = \frac{3}{2r} - 2 - r, \quad (12)$$

$$\frac{54}{25}g_{\text{NL}} = -\frac{9}{r} + \frac{1}{2} + 10r + 3r^2, \quad (13)$$

which lead to the following consistency relation,

$$g_{\text{NL}} = \frac{1}{54} \left[54f_{\text{NL}}^2 - 60f_{\text{NL}} - 125 - (9f_{\text{NL}} + 5) \sqrt{36f_{\text{NL}}^2 + 120f_{\text{NL}} + 250} \right]. \quad (14)$$

For $f_{\text{NL}} \gg 1$ ($r \ll 1$), this relation is approximated as [18]

$$g_{\text{NL}} = -\frac{10}{3}f_{\text{NL}} - \frac{575}{108} + \mathcal{O}(r). \quad (15)$$

Thus, the present constraint on f_{NL} , $-8.9 < f_{\text{NL}} < 14.3$ (95% CL), leads to $0.08 < r \leq 1$ (95% CL). The allowed region of g_{NL} is predicted to be $-53 < g_{\text{NL}} < 2$ (95% CL).

2.2 Modulated reheating

In the modulated reheating model, the decay rate of the inflaton Γ depends on some light field called modulus σ . After inflation, the inflaton starts the oscillation around its minimum. When the potential around the minimum is well approximated by a quadratic type, the energy density of the inflaton oscillation decays in proportional to a^{-3} (a : the

scale factor) and hence the oscillation behaves like a non-relativistic matter. On the other hand, the energy density of the Universe after reheating is dominated by radiation, whose energy density decays in proportional to a^{-4} . Thus, the fluctuations of the decay rate of the inflaton leads to the energy density (curvature) perturbations of the Universe. Assuming that the decay rate is much smaller than the Hubble parameter evaluated at the end of inflation ^{#2}, the final curvature perturbation is easily evaluated by the δN formalism as

$$\zeta = -\frac{1}{6}\frac{\Gamma_{\sigma}}{\Gamma}\delta\sigma_* + \frac{1}{12}\left(-\frac{\Gamma_{\sigma\sigma}}{\Gamma} + \frac{\Gamma_{\sigma}^2}{\Gamma^2}\right)\delta\sigma_*^2 + \frac{1}{36}\left(-\frac{\Gamma_{\sigma\sigma\sigma}}{\Gamma} + 3\frac{\Gamma_{\sigma}\Gamma_{\sigma\sigma}}{\Gamma^2} - 2\frac{\Gamma_{\sigma}^3}{\Gamma^3}\right)\delta\sigma_*^3, \quad (16)$$

which yields the non-linear parameters,

$$\frac{6}{5}f_{\text{NL}} = 6 - 6\frac{\Gamma\Gamma_{\sigma\sigma}}{\Gamma_{\sigma}^2}, \quad (17)$$

$$\frac{54}{25}g_{\text{NL}} = 36\left(2 - 3\frac{\Gamma\Gamma_{\sigma\sigma}}{\Gamma_{\sigma}^2} + \frac{\Gamma^2\Gamma_{\sigma\sigma\sigma}}{\Gamma_{\sigma}^3}\right). \quad (18)$$

It should be noticed that, when Γ linearly depends on σ and the other higher derivatives vanish, f_{NL} and g_{NL} are predicted to be $f_{\text{NL}} = 5$ and $g_{\text{NL}} = 100/3$.

Let us consider the following σ dependence on Γ as a more concrete example [13],

$$\Gamma = \Gamma_0 \left(1 + \alpha\frac{\sigma}{M} + \beta\frac{\sigma^2}{M^2}\right), \quad (19)$$

where α and β are constants and M is some energy scale. The non-linear parameters are rewritten as

$$\frac{6}{5}f_{\text{NL}} \simeq 6\left(1 - \frac{2\beta}{\alpha^2}\right), \quad \frac{54}{25}g_{\text{NL}} \simeq 36\left(2 - \frac{6\beta}{\alpha^2}\right). \quad (20)$$

Thus, the present constraint $-8.9 < f_{\text{NL}} < 14.3$ (95% CL) leads to $-0.9 < \beta/\alpha^2 < 1.4$ (95% CL). In this example, the third derivative of Γ , $\Gamma_{\sigma\sigma\sigma}$, is negligible, which leads to the following consistency relation,

$$g_{\text{NL}} = 10f_{\text{NL}} - \frac{50}{3}. \quad (21)$$

Therefore, g_{NL} is predicted to be $-106 < g_{\text{NL}} < 126$ (95% CL).

2.3 Inhomogeneous end of hybrid inflation

In hybrid inflation, the inflationary phase are kept thanks to the positive effective mass squared m_{χ}^2 of the waterfall field χ and ends at the critical value ϕ_{cr} of the inflaton ϕ due to the tachyonic instabilities. Then, if the effective mass squared of the waterfall field

^{#2}This assumption does not necessarily hold in general. For instance, see [26].

depends not only on the inflaton but also on another light field σ , the critical value ϕ_{cr} also fluctuates, which leads to the perturbation of the duration of the inflation, that is, δN [27–31].

Following Ref. [29], let us consider a potential of the form,

$$V = \frac{\lambda}{4} \left(\frac{v^2}{\lambda} - \chi^2 \right)^2 + \frac{1}{2} g^2 \phi^2 \chi^2 + \frac{1}{2} m_\phi^2 \phi^2 + \frac{1}{2} f^2 \sigma^2 \chi^2 + \frac{1}{2} m_\sigma^2 \sigma^2, \quad (22)$$

which yields the effective mass squared of the waterfall field m_χ^2 and the critical value of the inflaton ϕ_{cr} as follows,

$$m_\chi^2 = -v^2 + g^2 \phi^2 + f^2 \sigma^2, \quad \phi_{\text{cr}} = \frac{\sqrt{v^2 - f^2 \sigma^2}}{g}. \quad (23)$$

Here f, g , and λ are coupling constants and v is some scale related to the vacuum expectation value. The total duration (e-folding number) of the inflation is easily estimated as

$$N = -\frac{1}{M_{\text{Pl}}^2} \int_{\phi_*}^{\phi_{\text{cr}}} \frac{V}{V_\phi} d\phi, \quad (24)$$

which fluctuates due to the perturbation of ϕ_{cr} and generates the curvature perturbations,

$$\begin{aligned} \zeta = & \frac{\partial N}{\partial \phi_{\text{cr}}} \frac{d\phi_{\text{cr}}}{d\sigma} \delta\sigma_* + \frac{1}{2} \left[\frac{\partial^2 N}{\partial \phi_{\text{cr}}^2} \left(\frac{d\phi_{\text{cr}}}{d\sigma} \right)^2 + \frac{\partial N}{\partial \phi_{\text{cr}}} \frac{d^2 \phi_{\text{cr}}}{d\sigma^2} \right] \delta\sigma_*^2 \\ & + \frac{1}{6} \left[\frac{\partial^3 N}{\partial \phi_{\text{cr}}^3} \left(\frac{d\phi_{\text{cr}}}{d\sigma} \right)^3 + 3 \frac{\partial^2 N}{\partial \phi_{\text{cr}}^2} \left(\frac{d\phi_{\text{cr}}}{d\sigma} \right) \left(\frac{d^2 \phi_{\text{cr}}}{d\sigma^2} \right) + \frac{\partial N}{\partial \phi_{\text{cr}}} \frac{d^3 \phi_{\text{cr}}}{d\sigma^3} \right] \delta\sigma_*^3. \end{aligned} \quad (25)$$

Then, the non-linear parameters are given by

$$\frac{6}{5} f_{\text{NL}} \simeq -M_{\text{Pl}} \sqrt{2\epsilon_{\text{cr}}} \frac{\phi_{\text{cr}}''}{\phi_{\text{cr}}'}, \quad (26)$$

$$\frac{54}{25} g_{\text{NL}} \simeq -(2\epsilon_{\text{cr}} - \eta_{\text{cr}}) \frac{18}{5} f_{\text{NL}} + 2M_{\text{Pl}}^2 \epsilon_{\text{cr}} \frac{\phi_{\text{cr}}'''}{\phi_{\text{cr}}'^3}, \quad (27)$$

where $\epsilon = M_{\text{pl}}^2 (V_\phi/V)^2/2$ and $\eta = M_{\text{pl}}^2 V_{\phi\phi}/V$ are the standard slow-roll parameters and we have omitted the contributions comparable to the slow-roll suppressed parameters. The subscript “cr” represents the values evaluated at the critical point ϕ_{cr} . From Eq. (23), we can easily evaluate the non-linear parameters as

$$\frac{6}{5} f_{\text{NL}} = \eta_{\text{cr}} \frac{v^2}{f^2 \sigma^2}, \quad \frac{54}{25} g_{\text{NL}} = 6\eta_{\text{cr}}^2 \frac{v^2}{f^2 \sigma^2}. \quad (28)$$

Then, the present constraint $-8.9 < f_{\text{NL}} < 14.3$ (95% CL) leads to $-10.7 < \eta_{\text{cr}} v^2/(f^2 \sigma^2) < 17.2$ (95% CL). We also have the following consistency relation,

$$g_{\text{NL}} = \eta_{\text{cr}} \frac{10}{3} f_{\text{NL}}. \quad (29)$$

Since $\eta_{\text{cr}} < 1$, g_{NL} is predicted to be $-30 < g_{\text{NL}} < 48$ (95% CL).

2.4 Inhomogeneous end of thermal inflation

In the inhomogeneous end of thermal inflation model [32], the effective coupling between a flaton field and the cosmic temperature, g , depends on some light field called modulus σ . During a mini-inflation phase, so-called thermal inflation, the flaton field is trapped at the false vacuum due to the thermal mass. When the temperature decreases down to the critical temperature depending on g , the flaton field starts to roll down to its VEV and the mini-inflation ends. Hence, the fluctuation of the effective coupling g leads to the fluctuations of cosmic e -folding number corresponding to the primordial curvature perturbations. The final curvature perturbation is given by

$$\zeta = \frac{1}{2} \frac{g_\sigma}{g} \delta\sigma_* + \frac{1}{4} \left(\frac{g_{\sigma\sigma}}{g} - \frac{g_\sigma^2}{g^2} \right) \delta\sigma_*^2 + \frac{1}{12} \left(\frac{g_{\sigma\sigma\sigma}}{g} - 3 \frac{g_{\sigma\sigma} g_\sigma}{g^2} + 2 \frac{g_\sigma^3}{g^3} \right) \delta\sigma_*^3, \quad (30)$$

where the subscript σ denotes the derivative in terms of σ . This yields the non-linearity parameters as

$$\frac{6}{5} f_{\text{NL}} = -2 + 2 \frac{g g_{\sigma\sigma}}{g_\sigma^2}, \quad (31)$$

$$\frac{54}{25} g_{\text{NL}} = 4 \left(2 - 3 \frac{g g_{\sigma\sigma}}{g_\sigma^2} + \frac{g^2 g_{\sigma\sigma\sigma}}{g_\sigma^3} \right). \quad (32)$$

In a similar way to the modulated reheating case, when g linearly depends on σ and the other higher derivatives vanish, the non-linearity parameters are predicted to be $f_{\text{NL}} = -5/3$ and $g_{\text{NL}} = 25/9$. When the effective coupling, g , has the following σ dependence as

$$g = g_0 \left(1 + \alpha \frac{\sigma}{M} + \beta \frac{\sigma^2}{M^2} \right), \quad (33)$$

the non-linearity parameters are given by

$$\frac{6}{5} f_{\text{NL}} \simeq 2 \left(-1 + \frac{2\beta}{\alpha^2} \right), \quad \frac{54}{25} g_{\text{NL}} \simeq 4 \left(2 - \frac{6\beta}{\alpha^2} \right). \quad (34)$$

In this case, the present constraint $-8.9 < f_{\text{NL}} < 14.3$ (95% CL) leads to $-2.2 < \beta/\alpha^2 < 4.8$ (95% CL).

In this model, we have also the consistency relation between the non-linearity parameters as

$$g_{\text{NL}} = -\frac{10}{3} f_{\text{NL}} - \frac{50}{27}, \quad (35)$$

where we have neglected $g_{\sigma\sigma\sigma}$ and this leads $-50 < g_{\text{NL}} < 28$ (95% CL).

2.5 Modulated trapping

When the inflaton has a non-trivial coupling to other fields, the resonant particle production can happen. Such particle production significantly decreases the speed of the inflaton due to the backreaction effects. Then, if such particle production process depends on another light scalar field σ through a coupling constant and/or a resonant point, the cosmic expansion is perturbed due to the perturbations of the light scalar field, which generates the curvature perturbations.

Following Ref. [33], let us consider the coupling between an inflaton ϕ and fermionic fields χ given by,

$$\mathcal{L}_{\text{int}} = -\frac{1}{2}\mathcal{N}(m - \lambda\phi)\bar{\chi}\chi, \quad (36)$$

where \mathcal{N} is the number of species of χ particles with the same mass. Here, m and λ are a coupling constant and the bare mass of χ , both of which are assumed to depend on another light field σ . When the inflaton reaches the particle production point $\phi_{\text{pp}} = m/\lambda$, the effective mass of χ vanishes so that χ particles are resonantly produced. The produced number density is estimated as

$$n_{\text{pp}} = \frac{\lambda^{3/2}}{2\pi^3} |\dot{\phi}_{\text{pp}}|^{3/2}, \quad (37)$$

where the subscript “pp” represents the quantities evaluated at the particle production time. Then, the equation of motion for the inflaton is modified as

$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV(\phi)}{d\phi} = \mathcal{N}\lambda n_{\text{pp}} \left(\frac{a}{a_{\text{pp}}}\right)^{-3} \Theta(t - t_{\text{pp}}). \quad (38)$$

In order to quantify the particle production effects, we define

$$\Delta\phi(t) \equiv \phi(t, \lambda \neq 0) - \phi(t, \lambda = 0), \quad (39)$$

which is easily evaluated from the equation of motion,

$$\Delta\phi = \int_{t_*}^{\infty} \Delta\dot{\phi} dt = \frac{\mathcal{N}\lambda n_{\text{pp}}}{9H_{\text{pp}}^2}. \quad (40)$$

Here we regard H and $dV(\phi)/d\phi$ almost constant because the duration of particle production is assumed to be short. According to the δN formalism, the final curvature perturbations are estimated as

$$\zeta = \Delta N^{(\lambda \neq 0)} = -H_* \frac{\Delta\phi}{\dot{\phi}_{\text{pp}}} = \frac{\lambda^{5/2}\mathcal{N}|\dot{\phi}_{\text{pp}}|^{1/2}}{18\pi^3 H_{\text{pp}}}, \quad (41)$$

which can be expanded with respect to $\delta\sigma_*$ as

$$\zeta = (\Delta N^{(\lambda \neq 0)})_{,\sigma} \delta\sigma_* + \frac{1}{2} (\Delta N^{(\lambda \neq 0)})_{,\sigma\sigma} (\delta\sigma_*)^2 + \frac{1}{6} (\Delta N^{(\lambda \neq 0)})_{,\sigma\sigma\sigma} (\delta\sigma_*)^3. \quad (42)$$

Though the general formulae for the non-linear parameters are a bit complicated, they are simply written in the case that both m and λ are proportional to σ ,

$$\frac{6}{5}f_{\text{NL}} = \frac{9}{5e\beta}, \quad (43)$$

$$\frac{54}{25}g_{\text{NL}} = \frac{27}{25e^2\beta^2}, \quad (44)$$

where β is the “efficiency factor” given by

$$\beta \equiv \frac{\text{Max}(\Delta\dot{\phi})}{|\dot{\phi}_{\text{pp}}|} = \frac{\mathcal{N}\lambda^{5/2}|\dot{\phi}_{\text{pp}}|^{1/2}}{6\pi^3eH_{\text{pp}}}. \quad (45)$$

Then, the present constraint $-8.9 < f_{\text{NL}} < 14.3$ (95% CL) leads to $\beta < 0.10$ (95% CL). We also have the following consistency relation,

$$g_{\text{NL}} = \frac{2}{9}f_{\text{NL}}^2. \quad (46)$$

Then, g_{NL} is predicted to be $0 < g_{\text{NL}} < 45$ (95% CL).

2.6 Velocity modulation

In Ref. [34], it was shown that if particles σ in the early universe have velocity fluctuation on large scales, their decay rate also acquires fluctuation through the fluctuation of the Lorentz factor and hence the curvature perturbation is generated via the same mechanism as the standard modulated reheating scenario. One of the scenario to realize such a situation is to assume that parent particle Σ which decays into the daughter particle σ has fluctuation of its mass δm_{Σ} due to its dependence on the light field having long wavelength fluctuations. Notice that because of the mass fluctuation δm_{Σ} , the resultant σ particles receive not only velocity modulation but also density perturbation $\delta\rho_{\sigma}$. As a result, the curvature perturbation generated at the time when σ decays consists of two components, namely, the one generated by the standard curvaton mechanism and the other generated by the velocity modulation. Since both two components originate from the same fluctuations δm_{Σ} , they are fully correlated to each other. Therefore, taking both the two effects mentioned above into account, the final curvature perturbation can be expanded in terms of δm_{Σ} as

$$\zeta = A_1 \frac{\delta m_{\Sigma}}{m_{\Sigma}} + \frac{1}{2}A_2 \left(\frac{\delta m_{\Sigma}}{m_{\Sigma}}\right)^2 + \frac{1}{6}A_3 \left(\frac{\delta m_{\Sigma}}{m_{\Sigma}}\right)^3, \quad (47)$$

where each coefficient A_i ($i = 1, 2, 3$) depends on the four parameters in the model, w_0 (equation of state parameter of σ at the time of its generation), Ω_{Σ} (density parameter of Σ at the time of its decay), w_{σ} (equation of state parameter of σ at the time of its decay)

and Ω_σ (density parameter of σ at the time of its decay). The expressions for the expansion coefficients A_i are lengthy (especially for A_3), we refer the readers to Ref. [34] for their explicit expressions. Assuming $w_0 = \frac{1}{3}$ and $\Omega_\Sigma \ll 1$ just for simplicity, the non-linearity parameters are given by

$$\begin{aligned}
f_{\text{NL}} &= \frac{20}{9(3w_\sigma(4w_\sigma - 1) + 1)^2\Omega_\sigma((3w_\sigma - 1)\Omega_\sigma + 4)} \left[w_\sigma(36w_\sigma(2w_\sigma - 1) + 5) + 1)(\Omega_\sigma - 3w_\sigma\Omega_\sigma)^2 \right. \\
&\quad \left. + 2(w_\sigma(3w_\sigma(24w_\sigma(6w_\sigma - 7) + 35) + 10) - 7)\Omega_\sigma + 2w_\sigma(9w_\sigma(8w_\sigma(6w_\sigma + 5) - 21) + 22) + 24) \right] \\
g_{\text{NL}} &= \frac{800}{243(3w_\sigma(4w_\sigma - 1) + 1)^3\Omega_\sigma^2(-3w_\sigma\Omega_\sigma + \Omega_\sigma - 4)^2} \\
&\times \left[(3w_\sigma - 1)(w_\sigma(3w_\sigma(27w_\sigma(4w_\sigma(96w_\sigma(2w_\sigma - 3) + 127) - 75) - 43) - 97) + 41)\Omega_\sigma^3 \right. \\
&\quad + 3(w_\sigma(3w_\sigma(9w_\sigma(w_\sigma(12w_\sigma(8w_\sigma(36w_\sigma + 19) - 413) + 2579) - 416) + 262) - 500) + 133)\Omega_\sigma^2 \\
&\quad + 2(w_\sigma(36w_\sigma(3w_\sigma - 1)(4w_\sigma - 1) + 1) + 1)(\Omega_\sigma - 3w_\sigma\Omega_\sigma)^4 \\
&\quad + 2(w_\sigma(3w_\sigma(3w_\sigma(3w_\sigma(12w_\sigma(12(37 - 12w_\sigma)w_\sigma + 133) - 5177) + 4570) - 728) + 674) - 517)\Omega_\sigma \\
&\quad \left. + 4(w_\sigma + 1)(9w_\sigma(w_\sigma(12w_\sigma(36w_\sigma(4w_\sigma + 9) - 181) + 265) - 2) + 209) \right]. \tag{49}
\end{aligned}$$

We can easily find that f_{NL} is at least $\mathcal{O}(1)$ for any choice of $(w_\sigma, \Omega_\sigma)$. For fixed value of Ω_σ , f_{NL} takes a maximum $\sim 15/\Omega_\sigma$ for $w \simeq 0.1$. Numerically, we find that $0.5 \lesssim g_{\text{NL}}/\tau_{\text{NL}} \leq 1$ for any $(w_\sigma, \Omega_\sigma)$. Thus, a consistency relation between f_{NL} and g_{NL} can be written as

$$g_{\text{NL}} = C_{\text{vm}} \frac{36}{25} f_{\text{NL}}^2, \quad 0.5 \lesssim C_{\text{vm}} \leq 1. \tag{50}$$

Using the Planck result $-8.9 < f_{\text{NL}} < 14.3$ (95% CL), the most conservative limit on g_{NL} in this model is given by $0 < g_{\text{NL}} < 294$ (95% CL).

3 Trispectrum as first signature of non-Gaussianity

In this section, we discuss interesting possibility in which trispectrum appears as a first signature of non-Gaussianities of the curvature perturbations. That is, we discuss the cases that f_{NL} is small, as shown by the Planck results, but g_{NL} can be significantly large. Note again that τ_{NL} is also small for single source because we have the relation $\tau_{\text{NL}} = 36f_{\text{NL}}^2/25$.

Generally speaking, in order to realize large g_{NL} while keeping f_{NL} small (that is, $N_{\sigma\sigma}/N_\sigma^2 = \mathcal{O}(1)$, $N_{\sigma\sigma\sigma}/N_\sigma^3 \gg 1$), we can take two options. The first option is to consider large third derivative of N , that is, $N_{\sigma\sigma\sigma}$. Such examples include curvaton with a self coupling, modulated reheating model with the non-trivial(cubic) dependence of a decay rate Γ on the modulus σ , and inhomogeneous end of thermal inflation with the cubic dependence of a coupling g on the modulus σ . In these examples, as long as such cubic dependences

are extraordinarily large, g_{NL} can be significantly large, which is quite manifest from the formulae given in the previous section.

The second option is to make the first derivative N_σ accidentally small,^{#3} which leads to large g_{NL} unless $N_{\sigma\sigma\sigma}$ is also suppressed. It should be noticed that the second derivative $N_{\sigma\sigma}$ needs to be mildly suppressed to keep f_{NL} of the order of unity. Such accidentally small N_σ can be obtained by tuning model parameters. For example, in the modulated decay of the curvaton model [35–37], (essentially) two model parameters (r and $\Gamma\Gamma_{\sigma\sigma}/\Gamma_\sigma^2$) appear. By taking their adequate combination, N_σ ($N_{\sigma\sigma}$) is significantly (mildly) suppressed, which leads to large g_{NL} with small f_{NL} without resorting to $N_{\sigma\sigma\sigma}$.

Such a situation also happens in the case that the same light field σ contributes to the curvature perturbations multiple times in different ways. For example, in light field models discussed in the previous section, the fluctuations of a light field other than the inflaton are converted to the curvature perturbations. However, except the curvaton (and the velocity modulation), the evolution of such a light field is followed only until the conversion and the subsequent evolution is simply assumed to be negligible. However, generally speaking, such a modulus has only weak (gravitationally suppressed) interactions so that it is long-lived and can easily contribute to the energy density of the Universe at late times. That is, it is probable that a modulus in a light field model also behaves like a curvaton at late epoch, which becomes another origin of the curvature perturbations. In this setting, the curvature perturbations ζ consists of two parts. The first part comes from each light field model contribution and the second part arises from the curvaton contribution,

$$\zeta = \zeta_{\text{light}} + \zeta_{\text{cur}}. \quad (51)$$

It should be noticed that both contributions ζ_{light} and ζ_{cur} arise only from the same perturbations $\delta\sigma_*$. Then, when the linear and quadratic terms cancel adequately but the cubic term does not, f_{NL} becomes small while significantly large g_{NL} can appear without resort to the cubic dependence of the couplings. This is exactly the second option we can take.

In this section, we discuss this kind of possibilities in detail. More concretely, we investigate, for each model, which conditions on model parameters are necessary to realize such a possibility.

3.1 Modulated Decay of the Curvaton

Now in this section, we consider a model where the decay rate of the curvaton is modulated due to fluctuations of some other light scalar field σ [35–37]. In this model, the curvature

^{#3}It should be noted that we also need to take into account the normalization of the power spectrum of the curvature perturbations.

perturbation, up to the third order, is given by

$$\begin{aligned}\zeta = & -\frac{r}{6} \frac{\Gamma_\sigma}{\Gamma} \delta\sigma_* - \frac{r}{72} \left[6 \frac{\Gamma_{\sigma\sigma}}{\Gamma} + (r^2 + 2r - 9) \frac{\Gamma_\sigma^2}{\Gamma^2} \right] \delta\sigma_*^2 \\ & - \frac{r}{1296} \left[36 \frac{\Gamma_{\sigma\sigma\sigma}}{\Gamma} + 18 (r^2 + 2r - 9) \frac{\Gamma_\sigma \Gamma_{\sigma\sigma}}{\Gamma^2} + (3r^4 + 10r^3 - 22r^2 - 54r + 135) \frac{\Gamma_\sigma^3}{\Gamma^3} \right] \delta\sigma_*^3,\end{aligned}\quad (52)$$

where r is the fraction of the energy density of the curvaton at the time of its decay and $\Gamma_\sigma = d\Gamma/d\sigma$ and so on. In this model, the non-linearity parameters are written as

$$f_{\text{NL}} = \frac{5}{2r} \left(3 - 2 \frac{\Gamma \Gamma_{\sigma\sigma}}{\Gamma_\sigma^2} \right) - 2 - r, \quad (53)$$

$$g_{\text{NL}} = \frac{25}{54r^2} \left[36 \frac{\Gamma^2 \Gamma_{\sigma\sigma\sigma}}{\Gamma_\sigma^3} + 18 (r^2 + 2r - 9) \frac{\Gamma \Gamma_{\sigma\sigma}}{\Gamma_\sigma^2} + 3r^4 + 10r^3 - 22r^2 - 54r + 135 \right]. \quad (54)$$

When $r \ll 1$, f_{NL} and g_{NL} are related as

$$g_{\text{NL}} = \frac{2}{3} \left(15 + 4 \frac{\Gamma^2 \Gamma_{\sigma\sigma\sigma}}{\Gamma_\sigma^3} - 18 \frac{\Gamma \Gamma_{\sigma\sigma}}{\Gamma_\sigma^2} \right) \left(-2 \frac{\Gamma \Gamma_{\sigma\sigma}}{\Gamma_\sigma^2} + 3 \right)^{-2} f_{\text{NL}}^2. \quad (55)$$

When the combination $\Gamma \Gamma_{\sigma\sigma}/\Gamma_\sigma^2$ is almost tuned to cancel the denominator in the coefficient of f_{NL}^2 in the right hand side, g_{NL} can be large even if f_{NL} is $\mathcal{O}(1)$ since the numerator is not necessarily canceled by such a choice of the functional form of Γ .

3.2 Modulated curvaton

Let us consider the case where the light field model contribution in Eq. (51) comes from the modulated reheating mechanism. Assuming the quadratic potential of the light field and neglecting $\Gamma_{\sigma\sigma\sigma}$, in such case the total curvature perturbation is given by

$$\begin{aligned}\zeta = & \left(\frac{2r}{3\sigma_*} - \frac{\Gamma_\sigma}{6\Gamma} \right) \delta\sigma_* + \left[\frac{1}{9\sigma_*^2} (3r - 4r^2 - 2r^3) - \frac{1}{12} \left(\frac{\Gamma_{\sigma\sigma}}{\Gamma} - \frac{\Gamma_\sigma^2}{\Gamma^2} \right) \right] \delta\sigma_*^2 \\ & + \left[\frac{4}{81\sigma_*^3} \left(-9r^2 + \frac{r^3}{2} + 10r^4 + 3r^5 \right) - \frac{1}{36} \left(\frac{2\Gamma_\sigma^3}{\Gamma^3} - \frac{3\Gamma_\sigma \Gamma_{\sigma\sigma}}{\Gamma^2} \right) \right] \delta\sigma_*^3.\end{aligned}\quad (56)$$

The non-linearity parameters are then given by

$$f_{\text{NL}} = \frac{5}{3} \left(-\frac{4r}{\sigma_*} + \frac{\Gamma_\sigma}{\Gamma} \right)^{-2} \left[3 \left(\frac{\Gamma_\sigma^2}{\Gamma^2} - \frac{\Gamma_{\sigma\sigma}}{\Gamma} \right) + \frac{4r}{\sigma_*^2} (3 - 4r - 2r^2) \right], \quad (57)$$

$$g_{\text{NL}} = -\frac{50}{3} \left(-\frac{4r}{\sigma_*} + \frac{\Gamma_\sigma}{\Gamma} \right)^{-3} \left[-\frac{2\Gamma_\sigma^3}{\Gamma^3} + \frac{3\Gamma_\sigma \Gamma_{\sigma\sigma}}{\Gamma^2} + \frac{8r^2}{9\sigma_*^3} (-18 + r + 20r^2 + 6r^3) \right]. \quad (58)$$

From the above expressions, we find that for fine-tuned parameters we simultaneously have small f_{NL} and large g_{NL} . For example, in case where Γ is given by Eq. (19) and the parameters are set to be $\alpha = 1$, $r = \Lambda^{-1}$ and $M/\sigma_* = \Lambda/4$ with a big parameter $\Lambda \gg 1$, we can have a situation where

$$f_{\text{NL}} \lesssim \mathcal{O}(1), \quad g_{\text{NL}} = \mathcal{O}(\Lambda), \quad (59)$$

by choosing β appropriately. Hence, we can simultaneously realize large g_{NL} and $f_{\text{NL}} \lesssim \mathcal{O}(1)$.

3.3 Inhomogeneous end of hybrid inflation and curvaton

In a similar way to the above discussion about the modulated curvaton mechanism, we consider the case where the fluctuations of the curvaton also induce inhomogeneous end of hybrid inflation discussed in 2.3. In this case, the total curvature perturbations are given by

$$\begin{aligned} \zeta = & \left(\frac{2r}{3\sigma_*} + \frac{1}{\eta_{\text{cr}}} \frac{f^2 \sigma_*}{g^2 \phi_{\text{cr}}^2} \right) \delta \sigma_* \\ & + \left[\frac{1}{9\sigma_*^2} (3r - 4r^2 - 2r^3) - \frac{1}{2M_{\text{Pl}}^2} \frac{f^4 \sigma_*^2}{g^4 \phi_{\text{cr}}^2} + \frac{1}{2\eta_{\text{cr}}} \frac{f^2 (v^2 + f^2 \sigma_*^2)}{g^4 \phi_{\text{cr}}^4} \right] \delta \sigma_*^2 \\ & + \left[\frac{4}{81\sigma_*^3} \left(-9r^2 + \frac{r^3}{2} + 10r^4 + 3r^5 \right) - \frac{1}{6M_{\text{Pl}}^2} \frac{f^4 \sigma_* (f^2 \sigma_*^2 + 3v^2)}{g^6 \phi_{\text{cr}}^4} + \frac{1}{6\eta_{\text{cr}}} \frac{f^4 \sigma_* (2f^2 \sigma_*^2 + 6v^2)}{g^6 \phi_{\text{cr}}^6} \right] \delta \sigma_*^3. \end{aligned} \quad (60)$$

Neglecting the Planck suppressed terms, the non-linearity parameters are given by

$$\begin{aligned} f_{\text{NL}} & \simeq \frac{5}{3} \left(\frac{2r}{3} + \frac{1}{\eta_{\text{cr}}} \frac{f^2 \sigma_*^2}{g^2 \phi_{\text{cr}}^2} \right)^{-2} \left[\frac{1}{9} (3r - 4r^2 - 2r^3) + \frac{1}{2\eta_{\text{cr}}} \frac{f^2 \sigma_*^2 (v^2 + f^2 \sigma_*^2)}{g^4 \phi_{\text{cr}}^4} \right], \quad (61) \\ g_{\text{NL}} & \simeq \frac{50}{9} \left(\frac{2r}{3} + \frac{1}{\eta_{\text{cr}}} \frac{f^2 \sigma_*^2}{g^2 \phi_{\text{cr}}^2} \right)^{-3} \\ & \times \left[\frac{4}{81} \left(-9r^2 + \frac{r^3}{2} + 10r^4 + 3r^5 \right) + \frac{1}{6\eta_{\text{cr}}} \frac{f^4 \sigma_*^4 (2f^2 \sigma_*^2 + 6v^2)}{g^6 \phi_{\text{cr}}^6} \right]. \end{aligned} \quad (62)$$

In denominators in the above expressions, both terms are positive definite and hence in order to realize large g_{NL} the both terms must be much smaller than unity at least. For $r < 1$, in the numerator of the expression of f_{NL} it is not possible to realize a cancellation to obtain the small f_{NL} . Hence, the small f_{NL} does not yield g_{NL} large enough to be detected in this scenario.

3.4 Inhomogeneous end of thermal inflation and curvaton

We can also consider the case where the light field contribution comes from the inhomogeneous end of thermal inflation considered in 2.4. In such case, we have

$$\begin{aligned}\zeta = & \left(\frac{2r}{3\sigma_*} + \frac{1}{2} \frac{g_\sigma}{g} \right) \delta\sigma_* + \left[\frac{1}{9\sigma_*^2} (3r - 4r^2 - 2r^3) + \frac{1}{4} \left(\frac{g_{\sigma\sigma}}{g} - \frac{g_\sigma^2}{g^2} \right) \right] \delta\sigma_*^2 \\ & + \left[\frac{4}{81\sigma_*^3} \left(-9r^2 + \frac{r^3}{2} + 10r^4 + 3r^5 \right) + \frac{1}{12} \left(-3 \frac{g_{\sigma\sigma}g_\sigma}{g^2} + 2 \frac{g_\sigma^3}{g^3} \right) \right] \delta\sigma_*^3.\end{aligned}\tag{63}$$

The non-linearity parameters are then given by

$$\begin{aligned}f_{\text{NL}} &= \frac{5}{3} \left(\frac{4r}{3\sigma_*} + \frac{g_\sigma}{g} \right)^{-2} \left[\frac{4r}{9\sigma_*^2} (3 - 4r - 2r^2) + \left(\frac{g_{\sigma\sigma}}{g} - \frac{g_\sigma^2}{g^2} \right) \right], \\ g_{\text{NL}} &= \frac{50}{27} \left(\frac{4r}{3\sigma_*} + \frac{g_\sigma}{g} \right)^{-3} \left[\left(-3 \frac{g_{\sigma\sigma}g_\sigma}{g^2} + 2 \frac{g_\sigma^3}{g^3} \right) + \frac{8r^2}{27\sigma_*^3} (-18 + r + 20r^2 + 6r^3) \right].\end{aligned}\tag{64}$$

Similar to the case of the modulated curvaton case, for example, in case where g is given by Eq. (33) and the parameters are set to be $\alpha = -1$, $r = \Lambda^{-1}$ and $M/\sigma_* = \Lambda/4$ with a large parameter Λ , we have

$$f_{\text{NL}} \lesssim \mathcal{O}(1), \quad g_{\text{NL}} = \mathcal{O}(\Lambda),\tag{65}$$

by choosing β appropriately. Hence, we can simultaneously realize large g_{NL} and $f_{\text{NL}} \lesssim \mathcal{O}(1)$.

3.5 Modulated trapping and curvaton

In case where a light field inducing modulated trapping mechanism behaves like curvaton in later dynamics, the curvature perturbation is given by

$$\begin{aligned}\zeta = & \left(\frac{2r}{3\sigma_*} + \frac{5e\beta}{6\sigma_*} \right) \delta\sigma_* + \left[\frac{1}{9\sigma_*^2} (3r - 4r^2 - 2r^3) + \frac{5e\beta}{8\sigma_*^2} \right] \delta\sigma_*^2 \\ & + \left[\frac{4}{81\sigma_*^3} \left(-9r^2 + \frac{r^3}{2} + 10r^4 + 3r^5 \right) + \frac{5e\beta}{48\sigma_*^3} \right] \delta\sigma_*^3.\end{aligned}\tag{66}$$

The non-linearity parameter are given by

$$f_{\text{NL}} = \frac{5}{3} (4r + 5e\beta)^{-2} \left[4r (3 - 4r - 2r^2) + \frac{45}{2} e\beta \right],\tag{67}$$

$$g_{\text{NL}} = \frac{25}{9} (4r + 5e\beta)^{-3} \left[\frac{16r}{3} (-18 + r + 20r^2 + 6r^3) + \frac{45}{2} e\beta \right].\tag{68}$$

In this scenario, due to the constraints on the parameters β and r as $0 < \beta < 1$ and $0 < r < 1$, it is hard to realize the large g_{NL} without violating the Planck constraint on f_{NL} .

4 Summary

Following the Planck 2013 results [4], we discussed models of generating local-type non-Gaussianity where a light field other than the inflaton plays a main role of generating the curvature perturbations. First, we showed the constraint on model parameters for each light field model, by introducing the constraint on local-type f_{NL} obtained in Planck 2013 results: XXIV. By using the consistency relation between the non-linearity parameters f_{NL} and g_{NL} for light field models as given in our previous paper [18], we also obtained the constraint on g_{NL} and found that for simple light field models g_{NL} does not become large enough to be detected even in forthcoming Planck data and other future experiments, within the Planck constraint on f_{NL} [38–42].

We also discussed the possibility of generating large g_{NL} in light field models within the Planck constraint on f_{NL} . We classified the possible models into two categories. One is to consider large third derivative of N , that is, $N_{\sigma\sigma\sigma}$ in δN formalism. Such examples include curvaton scenario with a self coupling and also modulated reheating (or inhomogeneous end of thermal inflation) scenario with non-trivial cubic dependence of a decay rate Γ (or a coupling g) on the modulus σ which does not appear in the non-linearity parameter f_{NL} . In such case, the consistency relation between f_{NL} and g_{NL} is no longer realized and hence we can realize the large g_{NL} within the Planck constraint. Another is to make the first derivative N_σ accidentally small, which leads to large g_{NL} unless $N_{\sigma\sigma\sigma}$ is also suppressed. Such accidentally small N_σ can be obtained in the case where the same light field σ contributes to the curvature perturbations multiple times in different ways. As examples, we considered the cases where a modulus in a light field model also behaves like a curvaton at late epoch. We found that when we consider the modulated reheating or the inhomogeneous end of thermal inflation scenario as a light field model by taking fine-tuned parameters we can realize the large g_{NL} due to the accidental cancellation. However, for the cases where the inhomogeneous end of hybrid inflation or the modulated trapping scenario is considered as a light field model we found it difficult to realize accidentally small N_σ and $N_{\sigma\sigma}$ and hence obtaining large g_{NL} within the Planck constraint is also difficult. Although information about the g_{NL} would be a useful tool to discriminate the light field models in future observations, it seems to be difficult to realize the measurable g_{NL} without fine-tuning.

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References

- [1] P. A. R. Ade *et al.* [Planck Collaboration], “Planck 2013 results. I. Overview of products and scientific arXiv:1303.5062 [astro-ph.CO].
- [2] P. A. R. Ade *et al.* [Planck Collaboration], “Planck 2013 results. XVI. Cosmological parameters,” arXiv:1303.5076 [astro-ph.CO].
- [3] P. A. R. Ade *et al.* [Planck Collaboration], “Planck 2013 results. XXII. Constraints on inflation,” arXiv:1303.5082 [astro-ph.CO].
- [4] P. A. R. Ade *et al.* [Planck Collaboration], “Planck 2013 Results. XXIV. Constraints on primordial non-Gaussianity,” arXiv:1303.5084 [astro-ph.CO].
- [5] K. Enqvist and M. S. Sloth, Nucl. Phys. B **626**, 395 (2002) [arXiv:hep-ph/0109214].
- [6] D. H. Lyth and D. Wands, Phys. Lett. B **524**, 5 (2002) [arXiv:hep-ph/0110002].
- [7] T. Moroi and T. Takahashi, Phys. Lett. B **522**, 215 (2001) [Erratum-ibid. B **539**, 303 (2002)] [arXiv:hep-ph/0110096].
- [8] G. Dvali, A. Gruzinov and M. Zaldarriaga, Phys. Rev. D **69**, 023505 (2004) [arXiv:astro-ph/0303591].
- [9] L. Kofman, arXiv:astro-ph/0303614.
- [10] N. Bartolo, S. Matarrese and A. Riotto, Phys. Rev. D **69**, 043503 (2004) [hep-ph/0309033].
- [11] D. H. Lyth and Y. Rodriguez, Phys. Rev. D **71**, 123508 (2005) [astro-ph/0502578].
- [12] M. Zaldarriaga, Phys. Rev. D **69**, 043508 (2004) [arXiv:astro-ph/0306006].
- [13] K. Ichikawa, T. Suyama, T. Takahashi and M. Yamaguchi, Phys. Rev. D **78**, 063545 (2008) [arXiv:0807.3988 [astro-ph]].
- [14] J. M. Maldacena, JHEP **0305**, 013 (2003) [astro-ph/0210603].
- [15] P. Creminelli and M. Zaldarriaga, JCAP **0410**, 006 (2004) [astro-ph/0407059].

- [16] Z. Huang and F. Vernizzi, arXiv:1212.3573[astro-ph.CO].
- [17] G. W. Pettinari, C. Fidler, R. Crittenden, K. Koyama and D. Wands, arXiv:1302.0832 [astro-ph.CO].
- [18] T. Suyama, T. Takahashi, M. Yamaguchi and S. Yokoyama, JCAP **1012**, 030 (2010) [arXiv:1009.1979 [astro-ph.CO]].
- [19] A. A. Starobinsky, JETP Lett. **42** (1985) 152 [Pisma Zh. Eksp. Teor. Fiz. **42** (1985) 124];
- [20] D. S. Salopek and J. R. Bond, Phys. Rev. D **42**, 3936 (1990).
- [21] M. Sasaki and E. D. Stewart, Prog. Theor. Phys. **95**, 71 (1996). [arXiv:astro-ph/9507001];
- [22] M. Sasaki and T. Tanaka, Prog. Theor. Phys. **99**, 763 (1998). [arXiv:gr-qc/9801017].
- [23] D. H. Lyth, K. A. Malik and M. Sasaki, JCAP **0505**, 004 (2005) [arXiv:astro-ph/0411220].
- [24] T. Suyama and M. Yamaguchi, Phys. Rev. D **77**, 023505 (2008) [arXiv:0709.2545 [astro-ph]].
- [25] M. Sasaki, J. Valiviita and D. Wands, Phys. Rev. D **74**, 103003 (2006) [arXiv:astro-ph/0607627].
- [26] Y. Watanabe and J. 'i. Yokoyama, arXiv:1303.5191 [hep-th].
- [27] F. Bernardeau and J. P. Uzan, Phys. Rev. D **67**, 121301 (2003) [arXiv:astro-ph/0209330].
- [28] F. Bernardeau, L. Kofman and J. P. Uzan, Phys. Rev. D **70**, 083004 (2004) [arXiv:astro-ph/0403315].
- [29] D. H. Lyth, JCAP **0511**, 006 (2005) [arXiv:astro-ph/0510443].
- [30] M. P. Salem, Phys. Rev. D **72**, 123516 (2005) [arXiv:astro-ph/0511146].
- [31] L. Alabidi and D. Lyth, JCAP **0608**, 006 (2006) [arXiv:astro-ph/0604569].
- [32] M. Kawasaki, T. Takahashi and S. Yokoyama, JCAP **0912**, 012 (2009) [arXiv:0910.3053 [hep-th]].
- [33] D. Langlois and L. Sorbo, JCAP **0908**, 014 (2009) [arXiv:0906.1813 [astro-ph.CO]].
- [34] K. Nakayama and T. Suyama, Phys. Rev. D **84**, 063520 (2011) [arXiv:1107.3003 [astro-ph.CO]].

- [35] D. Langlois and T. Takahashi, arXiv:1301.3319 [astro-ph.CO].
- [36] H. Assadullahi, H. Firouzjahi, M. H. Namjoo and D. Wands, arXiv:1301.3439 [hep-th].
- [37] S. Enomoto, K. Kohri and T. Matsuda, arXiv:1301.3787 [hep-ph].
- [38] V. Desjacques and U. Seljak, Phys. Rev. D **81**, 023006 (2010) [arXiv:0907.2257 [astro-ph.CO]].
- [39] J. Smidt, A. Amblard, C. T. Byrnes, A. Cooray, A. Heavens and D. Munshi, Phys. Rev. D **81**, 123007 (2010) [arXiv:1004.1409 [astro-ph.CO]].
- [40] D. M. Regan, E. P. S. Shellard and J. R. Fergusson, Phys. Rev. D **82**, 023520 (2010) [arXiv:1004.2915 [astro-ph.CO]].
- [41] T. Giannantonio, A. J. Ross, W. J. Percival, R. Crittenden, D. Bacher, M. Kilbinger, R. Nichol and J. Weller, arXiv:1303.1349 [astro-ph.CO].
- [42] T. Sekiguchi and N. Sugiyama, arXiv:1303.4626 [astro-ph.CO].